Enrollm.ent No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Differential and Integral Calculus

Subject Code: 4SC04MTC1 **Branch: B.Sc.(Mathematics,Physics)**

Semester: 4 **Date: 15/04/2017** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q.1	**Attempt the following questions:**	**(14)**
a)	Define: Gradient of the scalar field.	**(01)**
b)	For change of variable if the constant limits are of x then type of strip should be ____________ (a) horizontal (b) Vertical (c) Oblique (d) None of These	**(01)**
c)	True/False: Curvature of straight line is zero .	**(01)**
d)	$\int_0^{2\pi}\int_0^4 r\,dr\,d\theta = ...$ $(a)\ 16\pi \quad (b) 8\pi \quad (c)\ 4\pi \quad (d)\ none\ of\ these$	**(01)**
e)	Define: Unit vector.	**(01)**
f)	True/False: The gradient of a scalar point is always vector quantity.	**(01)**
g)	If $\varphi = xyz$, the value of $\lvert grad\ \varphi \rvert$ at the point (1,2,-1) is ________.	**(01)**
h)	True/False: Radius of curvature is not always positive.	**(01)**
i)	Define: solenoidal vector.	**(01)**
j)	$\int_1^2\int_0^x y\,dx\,dy = ...$ $(a)\ \frac{3x}{2} \quad (b)\ \frac{7}{6} \quad (c)\ \frac{6}{7} \quad (d)\ None\ of\ these$	**(01)**
k)	True/False: In partial differential equations number of independent variables are not more than one.	**(01)**
l)	True/False: In a Double integral outer limit is always constant.	**(01)**
m)	If $J = \frac{\partial(u,v)}{\partial(x,y)}$ & $J' = \frac{\partial(x,y)}{\partial(u,v)}$. Then JJ'=……… (a) 1 (b) -1 (c) 0 (d) None of these	**(01)**
n)	Define: Curvature.	**(01)**

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions **(14)**

a) Define Directional Derivative of function. Find the Directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 = 4$ at $(-1,2,1)$. **(05)**

b) Find the value of a if the vector $(ax^2y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2y^2)k$ has zero divergence. Find the $curl$ of the above vector which has zero divergent. **(05)**

c) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{(x+y+z)} dzdydx.$ **(04)**

Q.3 Attempt all questions **(14)**

a) Evaluate $\nabla e^{r^2}; where\ \vec{r} = xi + yj + zk\ \&\ r = |\vec{r}|.$ **(05)**

b) Evaluate $\iint_R x^2 dA$, where R is region bounded by $xy = 16$ and the lines $y = x, y = 0, x = 8.$ **(05)**

c) Eliminate the arbitrary function from the equation $z = xy + f(xy)$ **(04)**

Q.4 Attempt all questions **(14)**

a) Sketch the region of given integration, change the order of integration and evaluate the integral $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx.$ **(05)**

b) Solve $(y^2 + z^2)p - xyq + xz = 0.$ **(05)**

c) Show that the curve $y = x^4$ is concave upward at the origin. **(04)**

Q.5 Attempt all questions **(14)**

a) Evaluate $\iint_R (x+y)^2\, dxdy$, where R is the region bounded by $x + y = 0, x + y = 1,\ 2x - y = 0, 2x - y = 3,$ using transformation $u = x + y, v = 2x - y$ **(06)**

b) Define curl of a vector field. Show that a fluid motion is given by $v = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is Irrotational. **(05)**

c) Show that the curve $y = e^x$ is everywhere concave upward. **(03)**

Q.6 Attempt all questions **(14)**

a) Derive radius of curvature for cartesian curves. **(05)**

b) Define: Line integral. Find work done if $\vec{F} = 2x^2 j + 3xyk$ moving a particle in the xy-plane from(0,0) to (1,4) along the curve $y = 4x^2$. **(05)**

c) Find the equations of tangent plane & normal line at the point $(-2,1,-3)$ to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3.$ **(04)**

Q.7 Attempt all questions **(14)**

a) State Green's Theorem. Verify Green's Theorem for $\oint_c [(x^2-2xy)dx+(x^2y+3)dy]$, where C is the boundary of the region bounded by the parabola $y=x^2$ and line $y=x.$ **(09)**

b) Define Lagrange's equation. Solve $(y+z)p+(z+x)q=x+y.$ **(05)**

Q.8 Attempt all questions **(14)**

a) State Stokes's Theorem. Verify Stokes's theorem for the vector field $\vec{F}=(x^2-y^2)i+2xyj$ in the rectangular region in the xy-plane bounded by $x=-a, x=a, y=0, y=b.$ **(09)**

b) Define: Divergence. For which value of the component v_3 is $v=e^x\cos yi+e^x\sin yj+v_3k$ is solenoidal. **(05)**